THREE-DIMENSIONAL STATIC ANALYSIS OF MARINE CABLES

Somchai Chucheepsakul

Department of Civil Engineering

King Mongkut's Institute of Technology-Thonburi

Bangkok, Thailand

Silpachai Subwonglee

Highway Department, Ministry of Communications

Bangkok, Thailand

Computational Mechanics, Cheung, Lee & Leung (eds) © 1991 Balkema, Rotterdam. ISBN 90 5410 029 X

Three-dimensional static analysis of marine cables

Somchai Chucheepsakul
Department of Civil Engineering, King Mongkut's Institute of Technology, Thonburi, Bangkok, Thailand
Silpachai Subwonglee
Highway Department, Ministry of Communications, Bangkok, Thailand

ABSTRACT: A method to determine the static equilibrium configuration of marine cables in three - dimensional space is presented. In present approach, the known quantities are the top tension and the location at both ends of the cable while the total arc length and equilibrium configuration are to be determined. A functional is introduced in which the potential energy of stretching and virtual work done due to current loadings are included. A finite element method was developed to solve the problem and a numerical example is given.

1. INTRODUCTION

Marine cables deployed in deep water can be subjected to cross current which results in three dimensional configuration. The paper concerning three dimensional analysis of cable are found in Refs.[3, 4, 5, 6]. Most of the analysis assumed the total arc length as a known quantity, while the top tension is an unknown. In many cases the magnitude of top tension is specified either by the strength of cable or the lifting capacity. Thus, the problem of static analysis for the cable is to find the cable configuration and the total arc length when the top tension is given. Figure 1a shows such cable configuration in three dimensional space.

A variational method used previously for two dimensional cases (Ref. [2]) is extended to the present problem. Three dependent variables are used: the two horizontal coordinates of the centroidal line and the cable tension. To obtain these three unknowns, two variational equations and one equilibrium equation are used. An example of undersea cable given by Ref.[3] is used for demonstration.

2. EQUILIBRIUM EQUATIONS

The equilibrium equations for an infinitesimal segment of cable (Fig.1b) are [1, 3]

$$\frac{dT}{ds} = W_e \sin\phi - P_t \tag{1}$$

$$\frac{d\theta}{ds} = -P_b/T\cos\phi \tag{2}$$

$$\frac{d\phi}{ds} = (W_e \cos\phi - P_n)/T \tag{3}$$

in which T is tension in cable, W_e is the effective weight of cable, ϕ is the angle between the horizontal plane and the tangent vector $\hat{t}$ and θ is the angle between the projection of the tangent vector $\hat{t}$ in the horizontal plane. The geometric relation of the differential segment of cable is given by

$$\frac{dx}{ds} = \cos\phi\cos\theta, \quad \frac{dy}{ds} = \cos\phi\sin\theta, \quad \frac{dz}{ds} = \sin\phi \tag{4a, 4b, 4c}$$

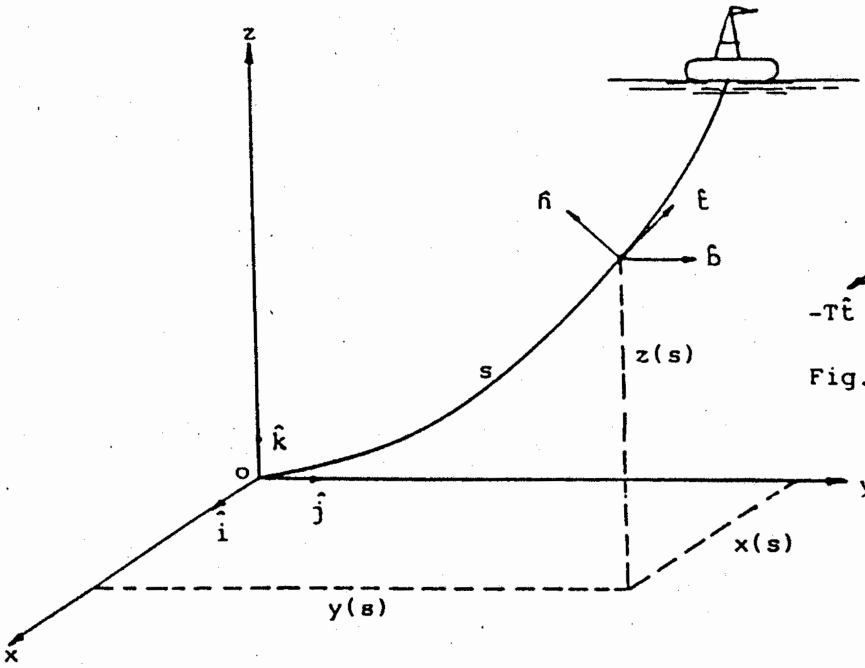

Fig. 1a Cable in equilibrium

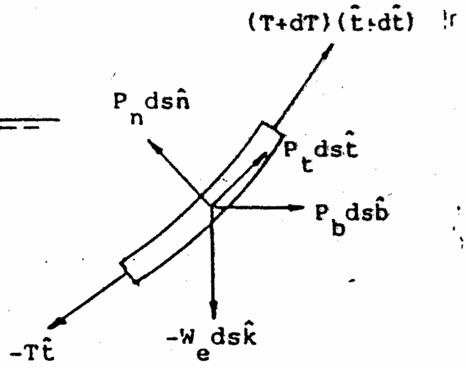

Fig. 1b Cable differential segment

The external forces P_t, P_b and P_n are the drag force due to current velocity in the horizontal plane which can be expressed as

$$P_t = \frac{1}{2}\rho_w C_{Dt}\pi D(V_x\cos\phi\cos\theta + V_y\cos\phi\sin\theta)^2 \tag{5}$$

$$P_b = \frac{1}{2}\rho_w C_{Dn} D(-V_x\sin\theta + V_y\cos\theta)^2 \tag{6}$$

$$P_n = \frac{1}{2}\rho_w C_{Dn} D(-V_x\sin\phi\cos\theta - V_y\sin\phi\sin\theta)^2 \tag{7}$$

where ρ_w is the density of sea water, C_{Dt} and C_{Dn} are tangential and normal drag coefficient, V_x and V_y are the components of current velocity in the horizontal plane.

Six independent variables T, x, y, z, ϕ and θ in Eqs.(1)-(4) are used to define any point s in the cable. These six equations can be solved numerically [3] with specified boundary conditions and cable length. The equilibrium configuration as well as tension along the cable length can be obtained. However if the cable length is not known at the beginning, the complexity arises and it is difficult to solve these equations. This difficulty can be overcome by introducing the variational formulation presented herein.

3. VARIATIONAL FORMULATION

An energy functional for analyzing marine cables in three dimensions can be written as

$$\Pi = \int_0^H \left(T\sqrt{1 + x'^2 + y'^2} - P_x . x - P_y . y\right) dz \tag{8}$$

where P_x and P_y are the distributed external forces per unit arc length acting along x and y directions respectively. These forces are defined as

$$P_x = (P_{tx} + P_{bx} + P_{nx})/\sin\phi \tag{9}$$

$$P_y = (P_{ty} + P_{by} + P_{ny})/\sin\phi \tag{10}$$

in which P_{tx}, P_{ty}, P_{bx}, P_{by}, P_{nx} and P_{ny} are the components of P_n, P_b and P_t in the x and y directions respectively. Using Eqs.(4) and Eqs.(5)-(7), one can write P_x and P_y as follows

$$\begin{aligned} P_x = \frac{1}{2}\rho_w D \{ & C_{Dt}\Pi(V_x\cos\phi\cos\theta + V_y\cos\phi\sin\theta)^2\cos\phi\cos\theta \\ & + C_{Dn}(-V_x\sin\theta + V_y\cos\theta)^2(-\sin\theta) \\ & + C_{Dn}(-V_x\sin\phi\cos\theta - V_y\sin\phi\sin\theta)^2(-\sin\phi\cos\theta)\}/\sin\phi \end{aligned} \tag{11}$$

$$\begin{aligned} P_y = \frac{1}{2}\rho_w D \{ & C_{Dt}\Pi(V_x\cos\phi\cos\theta + V_y\cos\phi\sin\theta)^2\cos\phi\sin\theta \\ & + C_{Dn}(-V_x\sin\theta + V_y\cos\theta)^2\cos\theta \\ & + C_{Dn}(-V_x\sin\phi\cos\theta - V_y\sin\phi\sin\theta)^2(-\sin\phi\sin\theta)\}/\sin\phi \end{aligned} \tag{12}$$

An expression for the cable tension at any position is obtained by integrating Eq.(1) from location z to the top end and using Eq.(4c), one gets

$$T = T_H + W_e(z - H) - \int_z^H \frac{P_t}{\sin\phi}\,dz \tag{13}$$

Normally P_t is small, then cable tension reduces to

$$T = T_H + W_e(z - H) \tag{14}$$

where T_H is the top tension and assumed to be known and H is the total depth.

4. NUMERICAL PROCEDURE

Three unknowns to be involved are x(z), y(z) and T(z). The stationary condition $\delta\Pi = 0$, which means vanishing of the variation due to x and the variation due to y, and Eq.(14) are used to solve this problem.

The prejection of element coordinates x(z) and y(z) on xz and yz planes can be written as

$$\begin{aligned} x(z) &= x_\ell + x_a \\ y(z) &= y_\ell + y_a \end{aligned} \tag{15}$$

The components x_ℓ and y_ℓ are linear, while x_a and y_a are nonlinear. The components x_a and y_a are approximated by a cubic polynomial in z. Thus,

$$\begin{Bmatrix} x_a \\ y_a \end{Bmatrix} = [N]\{d\} \tag{16}$$

where [N] is the matrix of shape function and {d} is the local degree of freedom.

The global equilibrium condition $\delta\Pi = 0$ yields the equilibrium equations for the entire system which is

$$\left\{ \frac{\partial\Pi}{\partial D_i} \right\} = \{0\} \tag{17}$$

This is a system of nonlinear equations. Then, by the Newton-Raphson iterative procedure, one can write the incremental equation as

$$[K_{NL}]\{\Delta D\} = -\{R\} \tag{18}$$

Consider the k^{th} element, the contribution to the square matrix $[K_{NL}]$ and to the vector $\{R\}$ are as follows

$$\left[\frac{\partial^2 \Pi_k}{\partial d_i \partial d_j} \right] = \int_0^h \frac{[N']^T T [N']}{(1 + x'^2 + y'^2)^{3/2}} \, dz \tag{19}$$

$$\left\{ \frac{\partial \Pi_k}{\partial d_i} \right\} = \int_0^h \left[\frac{[N']^T T}{(1 + x'^2 + y'^2)^{1/2}} \begin{Bmatrix} x' \\ y' \end{Bmatrix} - [N]^T \begin{Bmatrix} P_x \\ P_y \end{Bmatrix} \right] dz \tag{20}$$

where h is an element height. Eqs.(19) and (20) are evaluated by using Gaussian quadrature numerical integration. The iteration process begins with assuming an initial value of $\{D_o\}$. From Eq.(14) obtain tension T, then substitute T into Eqs.(19) and (20) and to obtain the new values $\{D_1\}$. Use $\{D_1\}$ as the initial value and repeat the iteration until convergence criterion is satisfied.

5. NUMERICAL EXAMPLE

An example of undersea buggy given by Ref.[3] is used for demonstration. The parameters used are listed in Table 1 and plan view of ship and buggy is shown in Fig. 2.

Table 1 Program Parameters

Water depth, m	183
Ship-to-buggy distance along seabed (R), m	229
Length of cable, m	305
Net weight of cable, N/m	12.3
Cable diameter, m	0.023
Surface current velocity in x-direction, km/hr	3.7
Normal drag coefficient	1.0
Tangential drag coefficient	0.005

By using 10 equal finite elements, the results of total cable length and the variation of cable tension for various horizontal angle ϕ_o (angle made by current with ship-to-buggy) are given in Table 2 and Fig. 3 respectively. A good agreements is obtained.

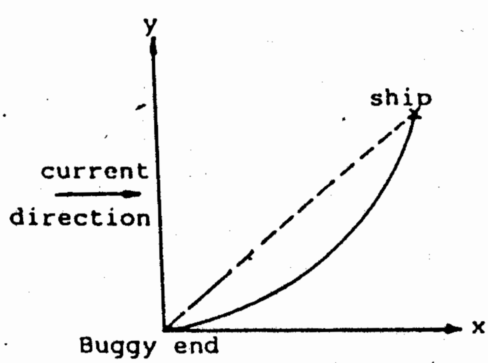

Fig. 2 Plan view of ship and buggy end

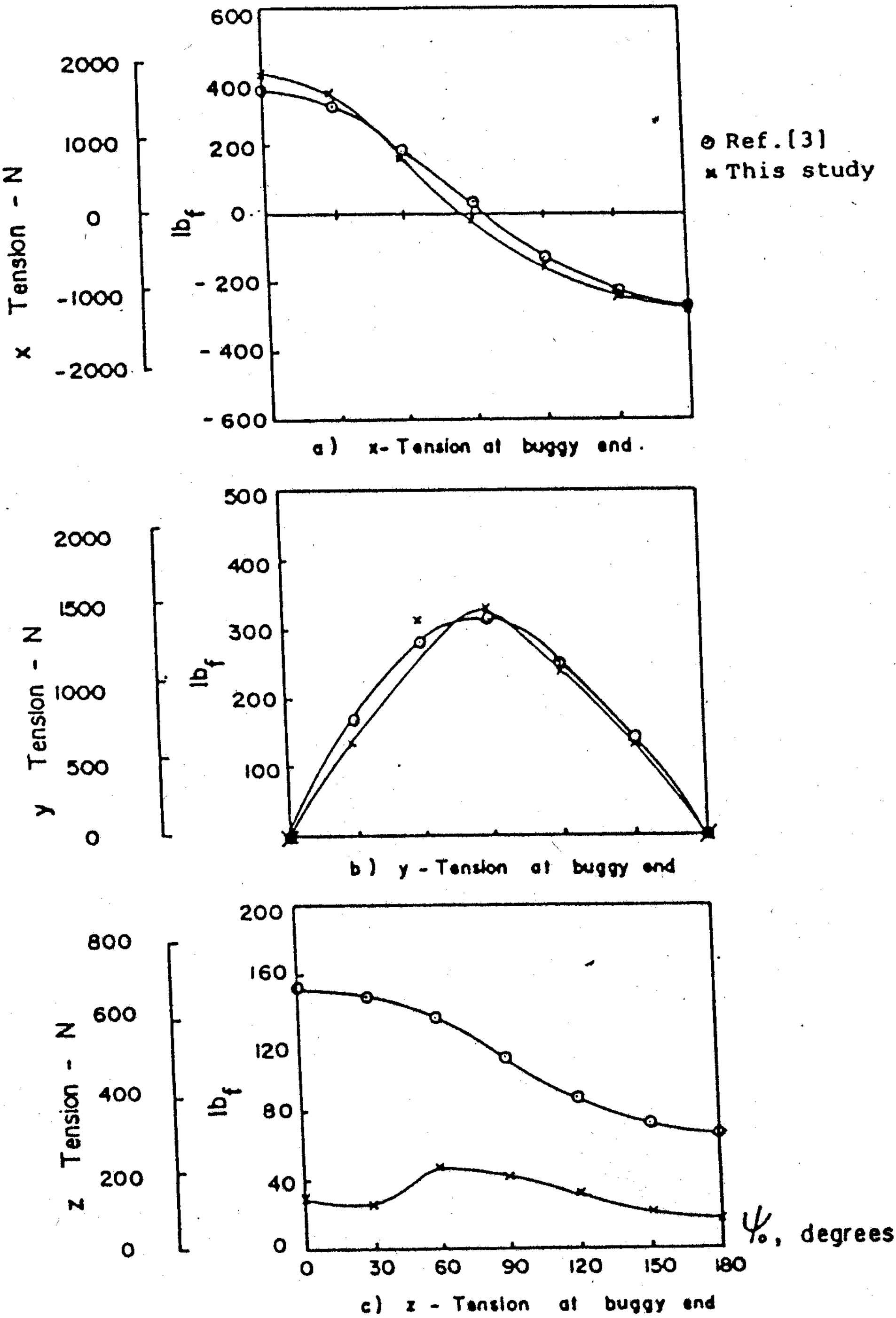

Fig. 3 Variation of tension with current direction

H=183 M , R=229 M

Table 2 Comparison of cable length, m

Ref.[3]	This study						
	$\psi_o = 0^\circ$	30°	60°	90°	120°	150°	180°
305	315	317	313	319	321	329	333

6. CONCLUSIONS

A variational formulation has been introduced for static analysis of marine cables in three dimensional space. A finite element method was developed to solve the problem. The method is suitable for marine cables having known top tension and unknown total cable length.

REFERENCES:

[1] Berteaux, H.O. (1976), Buoy Engineering, John Wiley & Sons, New York, NY, pp. 97-134.

[2] Chucheepsakul, S. and Huang, T. (1990), "Static Equilibrium of Marine Cables by a Variational Method," Proceedings of the First Pacific/Asia Conference on Offshore Mechanics and Arctic Engineering, Seoul, Korea, pp. 329-334.

[3] De Zoysa, A.P.K. (1978), "Steady - State Analysis of Undersea Cables," Ocean Engineering, Vol. 5, pp. 209-223.

[4] Felippa, C.A. (1974), "Finite Element Analysis of Three Dimensional Cable Structures," Proceedings of the International Conference on Computational Mothods in Nonlinear Mechanics, Austin, Texas, pp. 311-324.

[5] McAllister, T. and Leonard, J. (1986), "Program for Three-Dimensional Steady-State Analysis of Grapnel Lines," ASME Proceedings of the First OMAE Specialty Symposium on Offshore and Arctic Frontiers, pp. 285-290.

[6] Subwonglee, S., "Static Analysis of Cables in Two and Three Dimensions by a Variational Method," Master of Engineering Thesis, Dept. of Civil Eng., KMIT Thonburi, 1989.