PERFORMANCE ESTIMATION OF A PV WATER-PUMPING SYSTEM WITH UTILIZABILITY FUNCTION

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SUMMARY

A procedure using the concept of the utilizability function for estimating the long-term performance of direct-coupled photovoltaic water-pumping systems is presented. A modified utilizability function, based on radiation statistics, is developed, and the monthly-averaged daily amount of water is simply estimated.

KEY WORDS PV water-pumping system Utilizability function

INTRODUCTION

Water pumps powered by photovoltaics are an important alternative in rural areas remote from the grid. They are often used for both agricultural and domestic purposes. The concept of the utilizability function has been developed to better estimate PV water-pump performance.

Solar-radiation utilizability is defined as the fraction of the incident total solar radiation that exceeds a specified radiation value called the critical radiation level. This approach was originally developed for predicting the long-term average performances of flat-plate solar collectors [1] of which the critical level is the radiation intensity at which thermal losses from the collector are equal to its thermal gains, i.e. the net useful heat gain is zero. The monthly average hourly utilizability ϕ is defined as

$$\phi = (1/N) \sum^{N} (I_T - I_c)^+ / \bar{I}_T \tag{1}$$

where N is the number of days in a month, I_T is the radiation in a particular hour, $\bar{I}_T$ is the monthly-averaged radiation in this particular hour and I_c is the critical radiation level. The monthly-averaged daily utilizability $\bar{\phi}$ is also defined as the sum for a month, over all hours and all days, of the solar radiation incident on the collectors that is over a critical level, divided by the monthly radiation. Thus

$$\bar{\phi} = \sum_{day} \sum_{hour} (I_T - I_c)^+ / (\bar{H} N) \tag{2}$$

where $\bar{H}$ is monthly-averaged daily radiation, and the product (HN) is the monthly radiation value.

The value $\bar{\phi}$ for a specific location and a given month can be determined by drawing a cumulative curve (as shown in Figure 1) from the values of I_T for all the hours of sunshine, and for all the hours of the month. The horizontal axis is the fractional time $F(X)$ during which the hourly radiation is lower than or equal to the value of I_T (or the radiation ratio X is lower than or equal to the value of X'). The shaded area above the corresponding critical radiation ratio X_c represents the average total collectable solar energy during the hour over the entire month which is $\bar{\phi}$. Thus the daily utilizability function for flat-plate collectors can be

$$\bar{\phi} = \int_{X_c}^{X_{max}} [1 - F(X')] \, dX' \tag{3}$$

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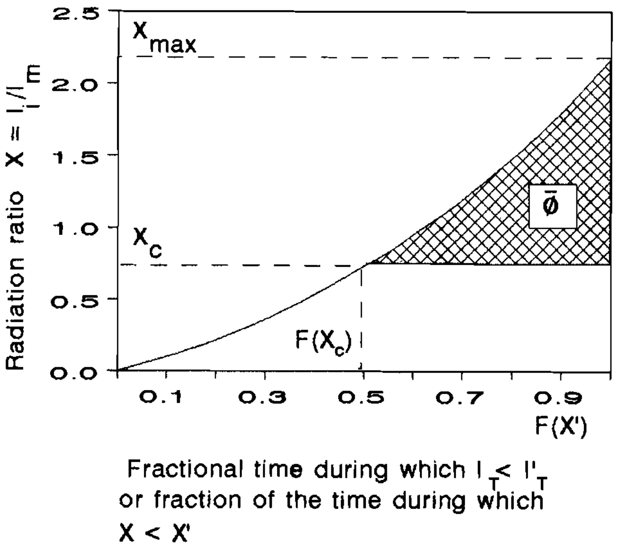

Figure 1. Cumulative frequency curve (shaded area represents utilizability fraction at X_c)

I'_T and X' denote specific values of I_T and X, respectively, $F(X')$ denotes the corresponding cumulative frequency distribution which is a fraction of the total time during which $X \leq X'$. For a specific location, the cumulative frequency curve can be curve-fitted, and the value $\bar{\phi}$ in equation (3) can be expressed simply in a functional form. For Bangkok, Thailand, $F(X)$ can be expressed as [2]

$$F(X) = a + bX + cX^2 + dX^3 \tag{4}$$

The values a, b, c and d are given in Table 1. The value of X_{max} can be estimated by taking $F(X_{max})$ to be 1.

With the value of $\bar{\phi}$, the monthly daily useful energy gain of solar collectors Q_u can be

$$Q_u = A_c \eta_o \bar{H} \bar{\phi} \tag{5}$$

where A_c and η_o are the collector area and the collector optical efficiency.

Many authors [3–5] have developed the utilizability concept for analysing the performance of photovoltaic systems. Siegel *et al.* [3] defined the critical level as the radiation intensity at which the electrical output matches the load, and the value ϕ represents the fraction of electrical production which exceeds the load. Clark *et al.* [4, 5] evaluated the utilizability function on a monthly-averaged hourly basis and extended the method to accommodate loads that vary from hour to hour. The technique has been used for predicting the long-term average performance of photovoltaic systems having storage batteries and subject to any diurnal load profile.

For any photovoltaic water-pumping system, the total load on the system depends on the total head difference between the stations, friction loss in the lines, and the pump performance. Therefore water can be pumped to any level when the electrical production from the photovoltaic array exceeds the load. More radiation over the critical level means a greater volume of water can be pumped. The problem is quite different from that of photovoltaic systems with storage, where the excess of the load will be stored in the batteries.

In this paper, a modified utilizability function for predicting the long-term average performance of photovoltaic arrays coupled directly to water-pumping systems is presented.

AVERAGE LONG-TERM PERFORMANCE

For the ith hour of the day, the average electrical output of the photovoltaic array is

$$E_i = A\eta I_i(CF) \quad \text{megajoules per hour} \tag{6}$$

Table 1. Values of constants a, b, c and d in equation (4)

Month	a	b	c	d	R^2	SEE	Order
Jan., Feb.	− 0·01	0·495	—	—	0·989	0·031	1
Mar., Dec.	0·011	0·421	0·032	—	0·99	0·029	2
	0·041	0·286	0·178	− 0·04	0·991	0·028	3
Apr., May June, July Aug.	0·056	0·469	—	—	0·997	0·014	1
Sept., Oct.	0·1501	0·3619	—	—	0·9795	0·0396	1
	0·0508	0·5709	− 0·077	—	0·9987	0·0101	2
	0·0715	0·4903	− 0·004	− 0·017	0·9991	0·0084	3
Nov.	0·0081	0·5521	− 0·054	—	0·998	0·0142	2
	0·0333	0·4489	0·0437	− 0·025	0·9985	0·0127	3

where A is the area of the photovoltaic in the array, I_i is the monthly-averaged hourly global radiation, η is the average efficiency of the array, and CF is the radiation correction factor for an inclined plane.

The monthly-averaged daily electrical output can now be evaluated from

$$E = \sum_{day} \sum_{hour} I_i A\eta(CF)/N \quad \text{megajoules per day} \tag{7}$$

which is also the power supplied to the pump to which the array is directly coupled. N is the number of days in a month.

The modified monthly-averaged daily utilizability $\bar{\phi}''$, which is the monthly average over all hours and all days for a month of global radiation when the radiation exceeds a critical level, is then defined:

$$\bar{\phi}'' = \sum_{day} \sum_{hour} I_i/N\bar{H}$$

or

$$\bar{\phi}'' = \sum_{day} \sum_{hour} (I_i - I_c)^+/N\bar{H} + \sum_{day} \sum_{hour} I_c/N\bar{H} \tag{8}$$

and

$$\bar{\phi}'' = \int_{X_c}^{X_{max}} [1 - F(X')]\,\mathrm{d}X + [1 - F(X_c)]X_c$$

$$= \bar{\phi} + [1 - F(X_c)]X_c \tag{9}$$

It can be seen that the value $\bar{\phi}''$ is the shaded area in the cumulative frequency curve in Figure 2. Now the critical radiation level I_c, and the critical radiation X_c are the radiation and the ratio at which the flow rate of water required equals zero. When the radiation exceeds this point, the water can be supplied.

The critical radiation level for water-pumping systems can be experimentally evaluated from the power supplied to the pump when water can be lifted to a designed level, but no flow condition is obtained. With the solar-cell-panel efficiency, the critical value can be estimated including the critical radiation ratio.

With the knowledge of $F(X)$, X_c and X_{max} at any location, the value of $\bar{\phi}$ and $\bar{\phi}''$ can be calculated from equations (3) and (9), respectively. The monthly-averaged daily power supplied to the pump from equation (7) when there is water flow can be rewritten as

$$E = A\eta\bar{H}\bar{\phi}''CF \quad \text{megajoules per day} \tag{10}$$

The hydraulic power P_h can be calculated from

$$P_h = \eta_h E \times 10^6/(t_d \times 3600) \quad \text{watts} \tag{11}$$

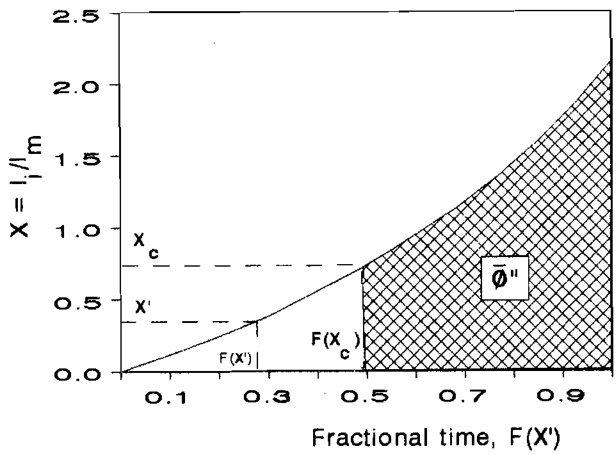

Figure 2. Cumulative frequency curve showing value of modified utilizability function $\bar{\phi}''$

where η_h is overall pump and motor efficiency and t_d is the number of working hours. Thus the daily amount of water Q can be estimated from

$$Q = 3600\,P_h t_d/(\rho g H_D), \quad \mathrm{m^3/d} \tag{12}$$

Where H_D is the head of water, in metres, at the pump.

EXAMPLE

The procedure already described has been developed to predict the performance of a water-pumping system at the King Mongkut's Institute of technology Thonburi, Bangkok [6].

The average values of the information inputs can be presented as follows:

Average head at pump H_D = 27·5 m water
Average efficiency η of the PV array = 9%
PV array area A = 8·0454 $\mathrm{m^2}$
Critical radiation I_c = 0·92 $\mathrm{MJ/m^2h}$
Average overall pump and motor efficiency η_h = 32%
Working hours in a day = 8

In January, for Bangkok, the average global radiation is 17 $\mathrm{MJ/m^2}$ per day, or the average hourly radiation is 1·41 $\mathrm{MJ/m^2h}$ (daytime = 12 h), and so the critical radiation ratio X_c will be

$$X_c = I_c/I_m = 0{\cdot}92/1{\cdot}41 = 0{\cdot}64$$

The values of X_{max} and $\bar{\phi}$ are calculated from equation (4), when $F(X_{max}) = 1$, and from equation (3), respectively, the results are

$$X_{max} = 2, \quad \bar{\phi} = 0{\cdot}5$$

With $X_c = 0{\cdot}64$ from equation (4), $F(X_c) = 0{\cdot}3$, then the value of $\bar{\phi}''$ from equation (9) can be evaluated as

$$\bar{\phi}'' = 0{\cdot}5 + (1 - 0{\cdot}3)0{\cdot}64 = 0{\cdot}96$$

With the radiation factor $CF = 1{\cdot}18$, the average power supplied to the pump is calculated from

$$E = 8{\cdot}0454 \times 0{\cdot}09 \times 17 \times 0{\cdot}96 \times 1{\cdot}18 = 13{\cdot}8 \text{ megajoules per day}$$

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$$\begin{aligned}\bar{\phi}'' &= \int_{X_c}^{X_{max}} [1 - F(X')]\,\mathrm{d}X + [1 - F(X_c)]X_c \\ &= \bar{\phi} + [1 - F(X_c)]X_c \end{aligned} \tag{9}$$

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With the knowledge of $F(X)$, X_c and X_{max} at any location, the value of $\bar{\phi}$ and $\bar{\phi}''$ can be calculated from equations (3) and (9), respectively. The monthly-averaged daily power supplied to the pump from equation (7) when there is water flow can be rewritten as

$$E = A\eta\bar{H}\bar{\phi}''CF \quad \text{megajoules per day} \tag{10}$$

The hydraulic power P_h can be calculated from

$$P_h = \eta_h E \times 10^6/(t_d \times 3600) \quad \text{watts} \tag{11}$$

Table 2. Simulated values of $\bar{\phi}''$, P_h and average daily amount of water Q in each month at KMITT, Bangkok

Month	$\bar{H}$	I_m	X_{max}	X_c	$F(X_c)$	$\bar{\phi}$	$\bar{\phi}''$	CF	E	P_h W	Q (est)*
Jan.	17·0	1·41	2	0·65	0·30	0·50	0·96	1·18	13·8	154	16·4
Feb.	18·6	1·55	2	0·59	0·27	0·54	0·97	1·12	14·7	163	17·4
Mar.	20·8	1·73	2	0·53	0·24	0·59	0·99	1·04	15·5	172	18·4
Apr.	20·4	1·70	2·01	0·54	0·31	0·51	0·88	0·97	12·6	140	15·0
May	19·3	1·61	2·01	0·57	0·32	0·49	0·87	0·92	11·2	125	13·3
June	17·4	1·45	2·01	0·63	0·35	0·45	0·86	0·91	9·8	109	11·6
July	18·1	1·51	2·01	0·61	0·34	0·46	0·86	0·92	10·4	116	12·3
Aug.	16·8	1·40	2·01	0·66	0·36	0·43	0·85	0·96	9·9	110	11·8
Sept.	15·3	1·28	3·56	0·72	0·42	0·37	0·79	1·02	8·9	99	10·6
Oct.	14·6	1·22	3·56	0·76	0·44	0·35	0·78	1·08	8·9	99	10·5
Nov.	14·7	1·23	3·79	0·75	0·39	0·18	0·63	1·13	7·6	85	9·0
Dec.	16·4	1·37	2	0·67	0·31	0·48	0·95	1·21	13·6	151	16·2

*(est) = estimated by this report

the hydraulic power P_h is

$$P_h = 0{\cdot}32 \times 13{\cdot}8 \times 10^6/(8 \times 3600) = 154 \quad \text{W}$$

and the average daily water amount Q is

$$\begin{aligned} Q &= 3600 \times 154 \times 8/(1000 \times 9{\cdot}81 \times 27{\cdot}5) \\ &= 16{\cdot}4 \text{ m}^3 \text{ per day.} \end{aligned}$$

The calculation is repeated for each month, and the results are presented in Table 2.

CONCLUSIONS

The utilizability concept is an effective tool for evaluation of the long-term performance of solar equipment. In this paper, a modified utilizability function $\bar{\phi}''$ has been presented for predicting the performance of a photovoltaic water-pumping system in Bangkok. The monthly-averaged daily amount of water has been estimated. The procedure can be performed easily for other local PV water-pumping stations where solar radiation data are collected.

NOMENCLATURE

A = area of photovoltaic array, m^2
A_c = area of solar collector area, m^2
CF = correction factor for converting horizontal radiation value to that for inclined plane
E = monthly-averaged daily electrical output, MJ per day
E_i = average electrical output, MJ/h
$F(X')$ = cumulative frequency distribution, which is also fraction of total time during which $X \leqslant X'$
H = monthly-averaged daily radiation, MJ/m^2 per day
I_c = critical solar radiation level, MJ/m^{2}h
I_i = monthly-averaged hourly global radiation, MJ/m^{2}h
I_T = solar radiation in a particular hour, MJ/m^{2}h
$\bar{I}_T$ = monthly-averaged hourly global radiation, MJ/m^{2}h
I_m = average hourly radiation, $I_m = H/12$
N = number of days in a month
P_h = hydraulic power, W

Q = daily amount of water, m^3 per day
Q_u = daily useful energy gain of solar collector, MJ per day
t_d = number of working hours in a day, h
X = radiation ratio
X_c = critical radiation ratio
X_{max} = maximum radiation ratio
η_o = optical efficiency of collector
η_h = overall pump and motor efficiency
η = average efficiency of photovoltaic array
ϕ = monthly-averaged hourly utilizability function
$\bar{\phi}$ = monthly-averaged daily utilizability function
$\bar{\phi}''$ = modified monthly-averaged daily utilizability function

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