Static Analysis of Marine Cables via Shooting-Optimization Technique

By C. M. Wang,[1] H. F. Cheong,[2] Members, ASCE, and S. Chucheepsakul,[3] Associate Member, ASCE

Abstract: Previous studies by De Zoysa and Chucheepsakul on a three-dimensional static analysis of marine cables yielded contradictory results for an example problem. This work addresses this discrepancy and furnishes the correct expressions for the set of differential equations governing the cable problem. The so-called shooting-optimization method is proposed as a solution technique. De Zoysa's cable problem was resolved, and presented herein are the correct variations of the tension components at the bottom end of the cable with respect to different angles between the ocean current direction and the projection of the ship-to-buggy line onto the horizontal plane.

Introduction

The many applications of marine cables (such as in oceanographic research, hydrographic surveying, salvage, telecommunications, fishing, towing and offshore technology) require accurate analysis to predict their static and dynamic behavior. The analysis is challenging because of the difficulty in modeling the hydrodynamic forces (which depends on the prevailing physical conditions) and the need for a reliable and efficient solution technique for determining the cable profile and tensions.

At a recent Asian-Pacific Conference on Computational Mechanics held in Hong Kong, Chucheepsakul (1991) presented a variational (finite element) method for the three-dimensional static analysis of marine cables. Illustrating his method with a cable problem considered by De Zoysa (1978), he determined the variations of the end tension components with respect to different horizontal angles made by the current with the ship-to-buggy line. The results for the vertical tensile component are shown to be significantly different from those obtained previously by De Zoysa who used the shooting method. This puzzling discrepancy is the motivating factor for this study.

On closer examination of the problem, it was discovered that previously presented forms of the governing differential equations are not correct and thus the solutions obtained hitherto are erroneous. In view of the presence of modulus signs in the differential equations, Chucheepsakul's proposed variational approach cannot be used in its present form and appropriate modifications must be made to cater for these signs if it is to be used.

This paper presents the governing set of differential equations and the shooting-optimization technique (Wang and Kitipornchai 1992) for solution.

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[1]Sr. Lect. in Civ. Engrg., Nat. Univ. of Singapore, Kent Ridge, Singapore 0511.

[2]Assoc. Prof. and Head in Civ. Engrg., Nat. Univ. of Singapore, Kent Ridge, Singapore 0511.

[3]Assoc. Prof. in Civ. Engrg., King Mongkut's Institute of Technology, Thonburi, Bangkok, Thailand.

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De Zoysa's cable problem is resolved and the correct tension variations and cable profiles are presented.

GOVERNING DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

Referring to Fig. 1, the equilibrium equations of a differential segment of an immersed cable can be shown to be given by (Berteaux 1976)

$$\frac{dT}{ds} = W_e \sin\phi - P_t \qquad (1)$$

$$\frac{d\theta}{ds} = -\frac{P_b}{T\cos\phi} \qquad (2)$$

$$\frac{d\phi}{ds} = \frac{W_e\cos\phi - P_n}{T} \qquad (3)$$

in which T = tension in the cable; W_e = effective weight of cable; s = cable scope measured from the bottom end of the cable; ϕ = angle between the horizontal x-y plane and the tangent vector $\mathbf{e}_t$; and θ = angle between the x axis and the projection of the tangent vector $\mathbf{e}_t$ in the horizontal plane.

Defining the x axis in the direction of the current velocity V and using Wilson's model, the hydrodynamic force components in the three vector directions $\mathbf{e}_t$, $\mathbf{e}_b$, $\mathbf{e}_n$ (see Fig. 1), can be shown to be given by

$$P_t = \frac{1}{2}\rho_w C_{Dt}\pi D V^2 \cos\phi\cos\theta|\cos\phi\cos\theta| \qquad (4)$$

$$P_b = -\frac{1}{2}\rho_w C_{Dn} D V^2 \sin\theta|\sin\theta| \qquad (5)$$

$$P_n = -\frac{1}{2}\rho_w C_{Dn} D V^2 \sin\phi\cos\theta|\sin\phi\cos\theta| \qquad (6)$$

in which ρ_w = density of sea water; C_{Dt} and C_{Dn} = tangential and normal drag coefficients; and D = diameter of the cable. Note that (4)–(6) contain

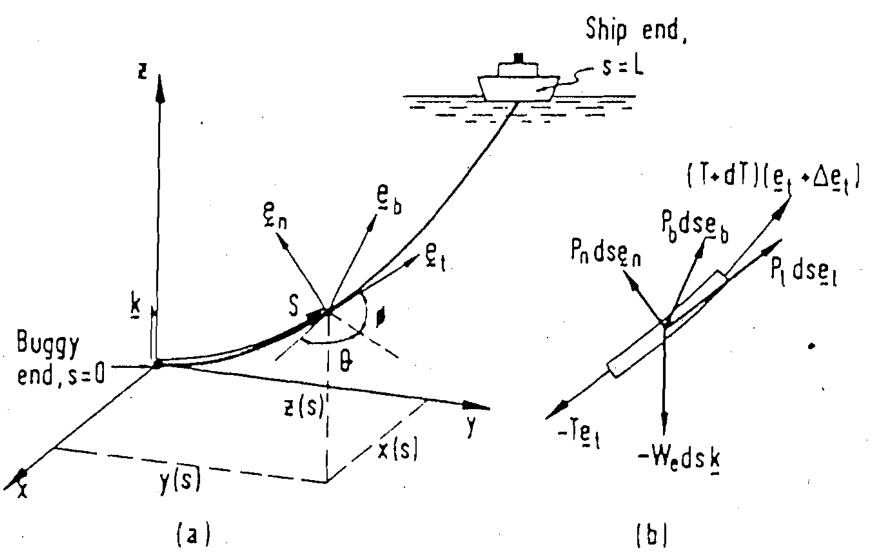

FIG. 1. Coordinate System for Marine Cable and Free Body Diagram of Cable Segment

modulus signs to account for the changing force directions in a three-dimensional situation. These modulus signs are absent in Berteaux's expression [see equations (4.90)–(4.92) on pp. 144–145 in Berteaux (1978)].

For convenience and generality, the following nondimensional parameters are introduced: $\bar{s} = s/H$; $\bar{x} = x/H$; $\bar{y} = y/H$; $\bar{z} = z/H$; $\bar{R} = R/h$; $\bar{L} = L/H$; $\bar{T} = T/(W_e H)$; $\bar{V} = V/\sqrt{gH}$; and $\alpha = \rho_w g DH/(2W_e)$. In the nondimensional parameters, R = projected length of the ship-to-buggy line on the seabed; H = depth of the seawater; L = cable length; and g = gravitational acceleration. The origin of the Cartesian coordinate system is taken at the bottom end of the cable, $s = 0$. Note that H was used to nondimensionalize the parameters instead of L because in general the depth of the sea is a known quantity, while L may be an unknown parameter.

In view of these nondimensional parameters, (1)–(3) together with (4)–(6) and the initial conditions may be written as

$$\frac{d\bar{T}}{d\bar{z}} = 1 - \pi\alpha C_{Dt}\bar{V}^2 \cot\phi \cos\theta|\cos\phi\cos\theta| \qquad \bar{T}(0) = \zeta_1 \quad \ldots\ldots\ldots (7)$$

$$\frac{d\theta}{d\bar{z}} = \frac{\alpha C_{Dn}\bar{V}^2}{\bar{T}\cos\phi\sin\phi}\sin\theta|\sin\theta| \qquad \theta(0) = \zeta_2 \quad \ldots\ldots\ldots (8)$$

$$\frac{d\phi}{d\bar{z}} = \frac{\cot\phi}{\bar{T}} + \frac{\alpha C_{Dn}}{\bar{T}}\bar{V}^2\cos\theta|\sin\phi\cos\theta| \qquad \phi(0) = \zeta_3 \quad \ldots\ldots\ldots (9)$$

where ζ_1, ζ_2, and ζ_3 are, respectively, the unknown values of $\bar{T}$, θ, and ϕ at the seabed, $\bar{z} = 0$.

Moreover, from geometrical considerations, one obtains

$$\frac{d\bar{x}}{d\bar{z}} = \cot\phi\cos\theta \qquad \bar{x}(0) = 0 \quad \ldots\ldots\ldots (10)$$

$$\frac{d\bar{y}}{d\bar{z}} = \cot\phi\sin\theta \qquad \bar{y}(0) = 0 \quad \ldots\ldots\ldots (11)$$

$$\frac{d\bar{s}}{d\bar{z}} = \operatorname{cosec}\phi \qquad \bar{s}(0) = 0 \quad \ldots\ldots\ldots (12)$$

The terminal boundary conditions for such cable problems are usually given by

$\bar{x}(1) = \bar{X}$ = normalized x-coordinate of the cable's top end .... (13)

$\bar{y}(1) = \bar{Y}$ = normalized y-coordinate of the cable's top end .... (14)

and depending on whether the top tension force or the length of the cable is given, the following respective end condition is to be satisfied:

$\bar{T}(1) = F$ = tension force parameter at the top end of cable .... (15)

or

$\bar{s}(1) = \bar{L}$ = length parameter of the cable .... (16)

Shooting-Optimization Technique

The governing six first-order differential equations (7)–(12) together with the six known end conditions may be solved for $\bar{T}$, θ, ϕ, $\bar{x}$, $\bar{y}$, $\bar{s}$ as functions of $\bar{z}$. In such marine cable problems, the top tension force may be the known quantity, while the cable length is unknown or vice versa. In the former case, the cable solutions may be determined by considering the first five differential equations (7)–(11), since the variation of the cable scope $\bar{s}$ with respect to $\bar{z}$ is not needed (unless the length of the cable is to be determined). In the latter case, where the cable length L is known and the top tension is unknown, then either one considers all the six differential equations (7)–(12), or one opts for solving the first five differential equations subject to the satisfaction of the constraint due to (12), i.e.

$$\bar{L} - \int_0^1 \operatorname{cosec} \phi \, d\bar{z} = 0 \qquad (17)$$

The foregoing boundary value problem may be solved using the shooting-optimization technique. This technique is basicaly the shooting method, but instead of solving a set of algebraic equations formed from the difference (or error) between the prescribed and computed terminal boundary conditions, the error norms are minimized by an optimization algorithm. We present the method as applied to the cable problem, where we shall use the fact that only five differential equations need to be considered.

First, the differential equations are integrated forward using, say, the fourth-order Runge-Kutta algorithm. Specify the step size of integration h, and the initial values at $\bar{z}(0) = 0$: $\bar{x}(0) = \bar{y}(0) = 0$, $\bar{T}(0) = \zeta_1$; $\theta(0) = \zeta_2$; and $\phi(0) = \zeta_3$. For $n = 0, 1, 2, \ldots, (1/h - 1)$ do:

$$\bar{z}(n+1) = \bar{z}(0) + (n+1)h \qquad (18)$$

$$C_{i1}(n) = hf_i[\bar{z}(n), \bar{T}(n), \theta(n), \phi(n), \bar{x}(n), \bar{y}(n)] \qquad (19)$$

$$C_{i2}(n) = hf_i\left[\bar{z}(n) + \frac{h}{2}, \bar{T}(n) + \frac{C_{11}}{2}, \theta(n) + \frac{C_{21}}{2}, \phi(n) + \frac{C_{31}}{2}, \bar{x}(n) + \frac{C_{41}}{2}, \bar{y}(n) + \frac{C_{51}}{2}\right] \qquad (20)$$

$$C_{i3}(n) = hf_i\left[\bar{z}(n) + \frac{h}{2}, \bar{T}(n) + \frac{C_{12}}{2}, \theta(n) + \frac{C_{22}}{2}, \phi(n) + \frac{C_{32}}{2}, \bar{x}(n) + \frac{C_{42}}{2}, \bar{y}(n) + \frac{C_{52}}{2}\right] \qquad (21)$$

$$C_{i4}(n) = hf_i[\bar{z}(n) + h, \bar{T}(n) + C_{13}, \theta(n) + C_{23}, \phi(n) + C_{33}, \bar{x}(n) + C_{43}, \bar{y}(n) + C_{53}] \qquad (22)$$

$$\Omega_i(n+1) = \Omega_i(n) + \frac{1}{6}[C_{i1} + 2C_{i2} + 2C_{i3} + C_{i4}] \qquad (23)$$

where $\Omega_1 = \bar{T}$; $\Omega_2 = \theta$; $\Omega_3 = \phi$; $\Omega_4 = \bar{x}$; and $\Omega_5 = \bar{y}$.

Next, the sum of the L_1 error norms given by the difference in values of $\bar{x}$, $\bar{y}$, and $\bar{T}$, and the prescribed terminal boundary conditions of (13), (14), and (15) is minimized by any standard direct search-optimization technique. If the top tension is unknown and the cable length is known, then the error norm for $\bar{T}$ is substituted by taking the absolute quantity of the LHS of (17). The objective function for the optimization exercise is then given by either (a) For given top tension while cable length is unknown

$$\underset{\zeta_1,\zeta_2,\zeta_3}{\text{Min}}\ \Phi = |\bar{x}(1) - \bar{X}| + |\bar{y}(1) - \bar{Y}| + |\bar{T}(1) - \bar{F}| \quad (24)$$

or (b) For given cable length while top tension is unknown

$$\underset{\zeta_1,\zeta_2,\zeta_3}{\text{Min}}\ \Phi = |\bar{x}(1) - \bar{X}| + |\bar{y}(1) - \bar{Y}| + \left| \int_0^1 \operatorname{cosec} \phi \, d\bar{z} - \bar{L} \right| \quad (25)$$

It is clear that the desired value of Φ is zero for the solution. For the optimization algorithm, we have used the simplex method of Nelder and Mead (1964).

It should be remarked that the integration of the differential equations (with respect to $\bar{z}$ coordinate from the seabed to the surface) has the implicit constraint that the cable profile must lie in the positive domain of $\bar{z}$. This constraint is desirable, as the cable is expected to be above the seabed.

Solutions to Some Cable Problems

Berteaux's Two-Dimensional Cable Problem (1976)

Before solving De Zoysa's three-dimensional cable problem, we test the shooting-optimization method on a two-dimensional cable problem solved in Berteaux's (1976) book. Consider a surface buoy that is moored with a cable of diameter D = 6.35 mm, and submerged weight of W_e = 1.4628 N/m in a water depth of H = 2,600 m. The speed of the uniform current is V = 0.762 m/s. The density of seawater is taken to be ρ_w = 1,025 kg/m^3, and let the excursion X = 1,280 m, the coefficients C_{Dn} = 1.54 and C_{Dt} = 0.0154. The problem is to determine the cable length L, the angle ϕ at the bottom and top ends of the cable for a given tension of F = 11.1466 kN at the buoy.

Based on the preceding information, some of the input parameters for the shooting-optimization method are

$$\bar{V} = 0.0047712678 \quad (26a)$$

$$\alpha = 56744.66007 \quad (26b)$$

and the objective function is given by

$$\underset{\zeta_i}{\text{Min}}\ \Phi = \left| \bar{x}(1) - \frac{32}{65} \right| + |\bar{y}(1)| + |\bar{T}(1) - 2.930786058| \quad (27)$$

Using an integration step size of h = 0.05, the shooting-optimization method yields results which are tabulated in Table 1. The close agreement of these results with previously obtained ones by Berteaux (1976) and Chucheepsakul and Huang (1990) provides a confirmation to the validity and accuracy of the method. Berteaux used Wilson's tables of cable functions,

TABLE 1. Numerical Comparison of Two-Dimensional Cable Problem

(1)	Berteaux (1978) (2)	Chucheepsakul (1991) (3)	Present study (4)
Cable length (m)	3,030	3,016	3,013
ϕ (z = H) (degrees)	90.0	90.4	90.2
ϕ (z = 0) (degrees)	35.9	35.7	36.3

and Chucheepsakul used 20 elements for his variational (finite element) method. For this two-dimensional cable analysis, the variational method proposed by Chucheepsakul can be used because the terms in the modulus signs in the differential equations are positive for the entire length of cable.

De Zoysa's Three-Dimensional Cable Problem

A sea buggy on the seabed is to have its power supplied by a cable as shown by Figs. 1 and 2. The following set of information is given:

- Speed of uniform current, $V = 3.7$ km/hr
- Water depth, $H = 183$ m
- Ship-to-buggy distance along seabed, $R = 229$ m
- Length of cable, $L = 305$ m
- Net weight of cable, $W_e = 12.3$ N/m
- Cable diameter, $D = 0.023$ m
- Normal drag coefficient, $C_{Dn} = 1.0$
- Tangential drag coefficient, $C_{Dt} = 0.005$
- Density of seawater, $\rho_w = 1021$ kg/m^3

The problem is to determine the cable profile and tension components (T_x, T_y, T_z) at the buggy end for various angles ψ_0 between the x axis and the projected ship-to-buggy line onto the horizontal plane.

The specified terminal boundary conditions are

$$\bar{x}(1) = \bar{R}\cos\psi_0; \qquad \bar{y}(1) = \bar{R}\sin\psi_0 \qquad (28)$$

This means that the objective function to be minimized in the shooting-optimization techniques is defined as

$$\underset{\zeta_1,\zeta_2,\zeta_3}{\text{Min}}\ \Phi = |\bar{x}(1) - \bar{R}\cos\psi_0| + |\bar{y}(1) - \bar{R}\sin\psi_0| + \left|\int_0^1 \operatorname{cosec}\phi\, d\bar{z} - \bar{L}\right| \qquad (29)$$

in which $\zeta_1 = \bar{T}(0)$; $\zeta_2 = \theta(0)$; and $\zeta_3(0) = \phi(0)$.

The input parameters are

$$\bar{V} = 0.0242571165 \qquad (30a)$$

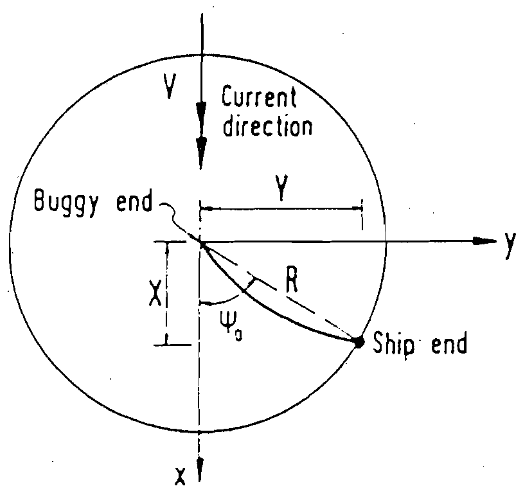

FIG. 2. Plan View of Ship and Buggy

$$\bar{R} = \frac{229}{183} \quad \text{(30b)}$$

$$\bar{L} = \frac{305}{183} \quad \text{(30c)}$$

$$\alpha = 1713.714882 \quad \text{(30d)}$$

Applying the shooting-optimization technique, the variations of the tension components at the buggy end are plotted in Fig. 3. Note that a convergence study shows that the integration step (for the Runge-Kutta algorithm) of $h = 0.05$ is sufficient for reasonably accurate solution. Comparing the results with those obtained by Chucheepsakul (1991) and De Zoysa (1978), which are shown in the inset, it can be seen that the tension components are not the same in magnitude, but the trends in the tension variations are somewhat similar.

It appears that the results of the previous researchers are erroneous. For example, when $\psi_0 = 90°$, their tension component in the x direction T_x is close to zero. The present calculations show that $T_x \approx 1{,}580$ N, which is a more reasonable solution, as one would expect that the cable's slope at the buggy end should not be perpendicular to the current direction. This condition is, however, implied in the previously obtained solution of $T_x \approx 0$. Furthermore, a rerun of Chucheepsakul's variational method software for the special cases of $\psi_0 = 0°$ and $\psi_0 = 180°$ (where the problem reduces to a two-dimensional one) yielded solutions close to the present results.

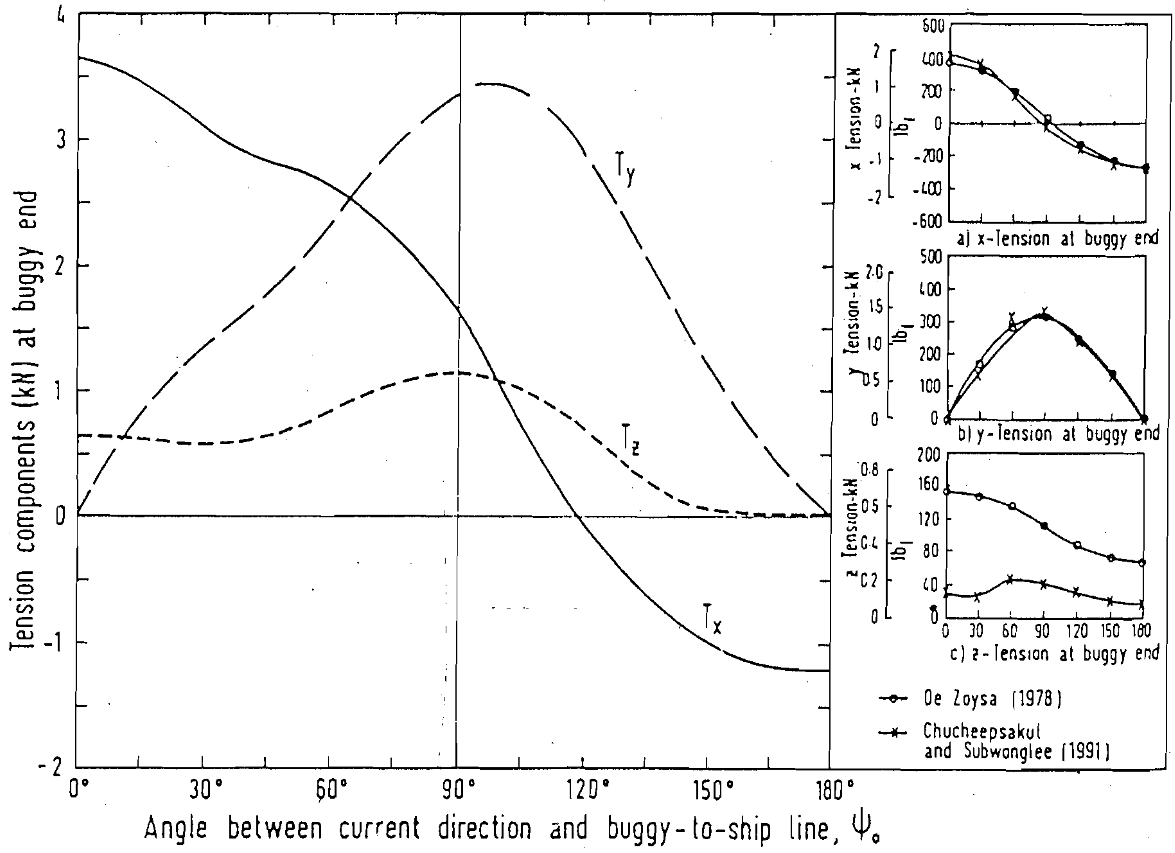

FIG. 3. Variations of Tension Components at Buggy End with Respect to Different Angles ψ_0

Concluding Remarks

The steady-state or static analysis of marine cables requires the solving of a set of nonlinear differential equations. As shown herein, this can be done using the shooting-optimization method. It is important to note that the differential equations should contain modulus signs to account for the changing force directions and Chucheepsakul's variational method should be modified to cater to these signs.

Although a uniform current velocity has been assumed in the problems considered herein, the shooting-optimization method can readily handle any variation of the velocity profile with respect to the water depth.

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